"Oh, Subtraction Hurts Me!"

by

Ed Rathmell

After a teacher asked me to talk with a second grade boy about subtraction facts, my goal was to determine how he was thinking to solve the problems. I soon found out that his strategy was to guess. He felt that he was off the hook as soon as he said any "answer." The answer might be incorrect, but so what, …he didn't really try. He had just guessed. Sometimes you guess right, sometimes you guess wrong.

But, his feelings became evident even before that. When I informed him that his teacher had asked me to talk with him about subtraction, he had a pained expression as he said, "Oh, subtraction hurts me!" That was one of my most uncomfortable interviews of the hundreds I have conducted. Subtraction really did hurt him, …and it was obvious during each problem I presented. He knew that he didn't understand, and guessing was the only strategy that he had figured out yet.

Primary grade teachers have students like this every year. Unfortunately, the materials they have do not provide the kind of help that is needed for these students. Textbooks simply don't provide enough time and experiences for most students to make sense and to develop flexibility and fluency, that is, to deeply understand. The pandemic has exacerbated this problem for all students, but especially for students who struggle learning math.

Teachers have had an impossible job to recreate ways to teach math this past year. It is not their fault. How do we help students make sense when we can't be in the same room? How do we keep them actively involved? How do we know what they are thinking? How do we help them make connections? How do we know what they have learned? How do we know if some students are being left behind? How do we prepare them for success in school math next year? …

It doesn't make any difference if students are in our classroom, on-line, homeschooling, or on vacation. There are no shortcuts. They will not understand unless they make sense of the concepts and the reasoning strategies they can use in everyday life. Drill and practice seem like the best solution to many people, but over 75 years of research has clearly

demonstrated that there are no long-term effects for most students. The focus is on the answer, not on how you can get the answer. Even as early as the 1940s, Brownell found that about 40% of all students did not get any immediate effect from drill. Long-term effects were almost non-existent. And drill and practice did nothing, for any student, to promote what we now call number sense. Research since then has repeatedly confirmed similar results.

Helping Students Understand Math

To make sense of math and be able to use it effectively, students need repeated experiences:

- to make sense of a variety of ways to represent each concept so they have a better opportunity to recognize when that concept can be used in everyday life,
- to make sense of a variety of reasoning strategies that can be used with each concept so they can efficiently use that concept with different numbers and in different contexts,
- with those representations and reasoning strategies so they can be used flexibly and fluently,
- using those concepts and reasoning strategies to solve problems they will encounter in a variety of everyday situations, and
- with frequent, but brief, low-stress formative assessments that provide feedback to both the students and the teacher.

These recommendations are all consistent with national and state mathematics standards. The one thing that differs slightly is the additional suggestion for repeated experiences. That comes from well-documented research on memory and learning. Mindless repetition of computational problems is not helpful. But the reality is that students do need repeated experiences with

- various ways to represent concepts,
- actions on those representations to make sense of various reasoning strategies, and
- using those representations and reasoning strategies to solve everyday problems,

to develop flexibility and fluency in their thinking. It simply takes time for students to internalize new thinking so they spontaneously use it in appropriate situations—much more time and experience than textbooks provide.

For example, after two weeks of brief daily lessons on using ten to add and subtract in the spring of grade 3, less than half of them spontaneously used that thinking when provided the opportunity. Even though they could explain that thinking when specifically asked, more than half of them resorted to much less efficient counting in other situations. Students need extended opportunities to make sense of new concepts and new thinking. They need time to make sense of the new thinking and internalize it to the point where they automatically use it when appropriate. Practice in the use of new thinking is essential, if we expect students to actually use that thinking. Just because symbolic drill and practice is not effective doesn't mean that repeated experiences with representations and reasoning strategies are not needed.

Meaningful Distributed Instruction

Thirty-three years ago I had the pleasure of observing Marsha Bachman's second grade math class in Grinnell. She used brief daily conceptual previews to help prepare her students for success with subtraction. These were not drill and not symbolic practice. They involved helping students make sense of ways to represent computational situations, then using actions on manipulatives, to make sense of reasoning that could be used to solve the problem. No symbols were written.

I visited her classroom to observe a lesson on addition. The first thing she did was to use base ten blocks to show 5 tens and 2 ones. Then she asked how many would be left if she subtracted 18. She then proceeded to ask questions as the students told her how to manipulate the tens and ones to answer the question. It surprised me that she started an addition lesson with a subtraction problem.

When I asked why she did that, she simply said, "I've found that it's much easier for the kids when we get to subtraction." Two weeks later after observing her students during the first day of instruction on subtraction, their understanding impressed me. By coincidence, I had just recently taught the same lesson using the same textbook, but with considerably more student confusion. That really got my attention!

Over the next few years, some of my undergraduate pre-service teachers and I tried similar approaches in action research studies. Altogether we covered about 20 different topics at grade levels ranging from K through grade 8. These conceptual previews led to overwhelming success. In every

instance students had at least a 20% achievement advantage over students without the previews. Several of my graduate students also did action research projects for their MA papers. In each case using conceptual previews enhanced achievement with similar results.

I want to highlight one of these studies. Tammy Boeckman, a sixth grade teacher in Ft. Dodge at the time, got amazing results. After using daily conceptual previews for fractions and decimals for the entire year (no symbolic practice), her students, including more than her share with learning problems, earned a class average score of over 90% on a very comprehensive fraction and decimal assessment—two years in a row. Nationwide, eighth grade students averaged about 20% lower on very similar National Assessment of Educational Progress (NAEP) items. Despite not practicing computation with fractions or decimals, her students performed over 10% higher on computation than eighth graders typically did on similar NAEP items.

Since the early 1990s, everything I have written, both articles and curriculum, has been based on using brief daily conceptual experiences to help students make sense and enhance their math achievement. And I stressed the importance of using similar approaches in each of my teacher education classes. Since no instructional materials are organized like that, I decided to retire from teaching so I could create what I had been promoting for years. For a thorough discussion of meaningful distributed instruction, see Chapter 5, Number and Operations: Organizing Your Curriculum to Develop Computational Fluency in Achieving Fluency: Special Education and Mathematics (NCTM, 2011).

www.thinkingwithnumbers.com
Now I have nearly completed an integrated and comprehensive collection of supplementary on-line lessons for addition and subtraction for students in grades K-3. They are currently over 1600 lessons that are:

- daily,
- supplementary,
- brief (about 5-minutes),
- conceptual,
- animated,
- planned with pauses after each question, and
- accompanied by brief formative assessments for each expected outcome.

The pauses are designed to provide students the opportunity to think, solve, explain, and discuss their solutions, …before one animated illustration of a reasoning strategy that could be used to solve the problem is presented. The lessons are designed for teachers to use a problem solving approach to instruction. And there are enough repeated experiences for students to have time to make sense and to develop flexibility and fluency.

Brief 5-item paper-and-pencil assessments will quickly inform teachers about student progress towards expected content outcomes. Additionally, there are on-line assessments for each reasoning strategy designed to inform teachers about progress with basic facts, but more importantly, about progress on actually using the reasoning strategy being assessed. Immediately after a class has used the on-line assessment for a designated reasoning strategy, teachers will have access to a list of students who are not yet using that strategy.

The topics include:

- Counting and Comparing,
- Numbers and Partitions,
- Exploring With Word Problems, and
- Reasoning Strategies.

The lessons are designed to help students make sense of different representations for the understandings and skills needed to use addition and subtraction. These representations include animated objects with five frames, ten frames, number lines, open number lines, tree diagrams, and

part-part-whole diagrams. The animations also illustrate the step-by-step thinking that can be used with each of the reasoning strategies.

The counting and comparing lessons, not only help students learn these skills, they also address all of the common students errors. This is the underlying knowledge needed to be successful with addition and subtraction. Most of this has been created for pre-K children.

The numbers and partitions lessons help students learn to use the structure of the five frame or ten frame to solve partition problems without counting. Students will understand part + part = whole and whole – part = other part in ways that connect their knowledge about addition and subtraction, something that students often lack. This lack is partially the result of subtraction language that does not connect to addition knowledge. Also thinking of subtraction only as "take away" does not help students make those connections.

There is a section on each of the Cognitively Guided Instruction problem structures. Most of these lessons have students solve or create a word problem. The others are animated illustrations of each problem structure.

They also provide enough lessons to help students make sense of different reasoning strategies while using addition and subtraction. Each of seven different strategies has at least four weeks of lessons followed by six weeks of practice for that thinking—far more than most students will need. These strategies include counting on, counting back, counting up to subtract, using tens to add and subtract, using known facts (including doubles) to add and subtract, using nice numbers, and changing the problem to one that is easier. Additionally, mental computation and estimation strategies include extending basic fact strategies to larger numbers, using front-end numbers, using nice numbers, changing the problem, using bounds, and using rounding.

These often overlooked reasoning strategies are crucial in helping students make progress in achievement. In Australia, bumps in achievement have been attributed to students developing new ways of thinking. For example, shortly after students learned to use ten to add and subtract, that group of students made a big jump in their achievement, as evidenced on tests. Those same reasoning strategies help with retention of basic facts. In three schools, all first and second grade students were interviewed to identify which reasoning strategies they could explain. Students who could explain a

strategy beyond counting dropped about 10% in performance over summer vacation. Students, who only counted, dropped over 50% in each school.

These on-line lessons are currently being provided free to anyone who registers as a parent so they can log in. You can examine the lessons and try them with your students by registering at:

www.thinkingwithnumbers.com

Please encourage primary grade classroom teachers, special education and resource teachers, and perhaps most important now, parents of young children in the primary grades to try our web site with their children. It is free; it just takes a commitment to spend 5 minutes a day with your child. More importantly, if the lessons are used daily for an extended period of time, it will make a difference in success with school math next fall.

Enjoy listening to your child. You can't believe how much fun it is to hear new, but confident, and unexpected explanations.

Ed Rathmell

Ed is an Emeritus Professor of Mathematics Education at the University of Northern Iowa. He is a former Iowa Council of Teachers of Mathematics President and long time member of the Board of Directors. He has created these on-line lessons at:
www.thinkingwithnumbers.com